

Full Marks : 70

Time : 3 hours

Answer Q. No. 1 which is compulsory and any five questions from the rest.

The figures in the right-hand margin indicate marks.

1. Answer all questions:

[2 x 10]

- (i) $F(n) = n^3 + n \log n$, then find $O(F(n))$. $= O(n^3 \log n)$
- (ii) $f(n) = n^3$, $g(n) = n^2$, then find $O(fg(n))$. $= n^6 = O(n^6)$
- (iii) $T(n) = 7T(n/3) + n^2$, then find $O(T(n))$. $O(n^2 \log n)$
- (iv) $T(n) = 2T(n/4) + \sqrt{n}$, then find $O(T(n))$. $O(\sqrt{n} \log n)$

(v) What is graph coloring problem? Give an example.

(vi) What is a flow network? What is skew symmetry property?

(vii) Construct a max heap tree with the data {C, O, M, P, U, T, E, R}.

(viii) What is memoization? How is it different from dynamic programming?

(ix) What is the basic difference between NP-hard and NP-complete problems?

(x) What do you mean by the convex hull of a given set of points? Give a suitable example.

2. (a) Differentiate between aggregate analysis and potential method of amortized analysis with suitable examples. [5]

(b) What are randomized algorithms? Explain the use of indicator random variables in randomized algorithms with a suitable example. [5]

3. (a) Explain the divide-and-conquer technique of analyzing algorithms taking merge sort as an example and derive its worst-case running time. [5]

(b) What are the elements of dynamic programming? Determine a longest common subsequence of $\langle 1, 0, 0, 1, 0, 1, 0, 1 \rangle$ and $\langle 0, 1, 0, 1, 1, 0, 1, 1, 0 \rangle$ using dynamic programming and show the table entries. [5]

LCS = 100110



→ fractional knapsack

$(1, 1, 1, 1, 1, 1, 0, 3, 3, 3), \sum w_i v_i = 24, \sum p_i v_i = 62.$

0/1 knapsack $(1, 1, 1, 1, 1, 1, 0), \sum w_i v_i = 22, \sum p_i v_i = 61.$

4. (a) Find an optimal solution to the Knapsack instance $n = 7, m = 24, w = \{2, 3, 5, 7, 1, 4, 6\}$ and $P = \{10, 5, 15, 7, 6, 18, 3\}$. [5]

(b) Explain why 0/1 Knapsack problem cannot be solved optimally using greedy method with a suitable example. [5]

5. (a) Explain the backtracking technique of analyzing algorithms with a suitable example. [5]

(b) Discuss Prim's algorithm to find the spanning tree of a given graph with a suitable example. [5]

6. (a) What is the basic difference between P, NP and NP-complete problems? Prove that the circuit-satisfiability problem belongs to the class NP. [5]

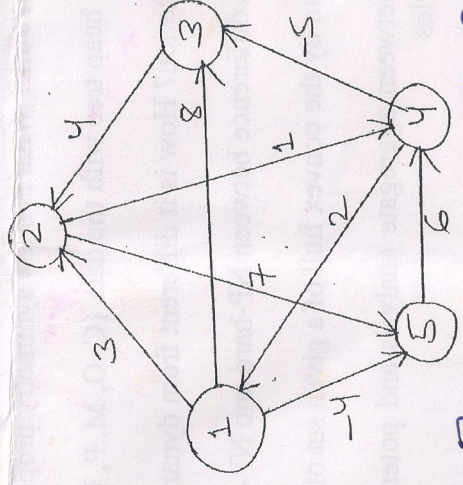
(b) Explain the Rabin-Karp string matching algorithm with a suitable example. [5]

7. (a) Explain how the vertex-cover problem can be solved using an approximation algorithm. [5]

(b) Discuss Graham's algorithm to find the convex hull of a given set of points. [5]

8. (a) Explain TVSP problem with a suitable example. [5]

(b) Run the Floyd-Warshall algorithm on the following weighted, directed graph and find the shortest path between each pair of vertices. Show the matrix $D^{(k)}$ that results from each iteration. [5]



$$D^{(1)} = \begin{bmatrix} 0 & 3 & 7 & 4 & -4 \\ \infty & 0 & 4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix}$$

$$D^{(2)} = \begin{bmatrix} 0 & 1 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix}$$

$$D^{(3)} = \begin{bmatrix} 0 & 3 & 8 & 1 & -4 \\ \infty & 0 & 4 & 0 & 7 \\ \infty & 4 & 0 & 8 & 8 \\ 2 & -5 & -8 & 0 & 6 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$$D^{(4)} = \begin{bmatrix} 0 & 3 & 8 & 1 & -4 \\ \infty & 0 & 4 & 0 & 7 \\ \infty & 4 & 0 & 8 & 8 \\ 2 & -5 & -8 & 0 & 6 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$$D^{(5)} = \begin{bmatrix} 0 & 3 & 8 & 1 & -4 \\ \infty & 0 & 4 & 0 & 7 \\ \infty & 4 & 0 & 8 & 8 \\ 2 & -5 & -8 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$$D^{(6)} = \begin{bmatrix} 0 & 3 & 8 & 1 & -4 \\ \infty & 0 & 4 & 0 & 7 \\ \infty & 4 & 0 & 8 & 8 \\ 2 & -5 & -8 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$